Classification of tight contact structures on small Seifert 3–manifolds with $e_0 \geq 0$

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Abstract We classify positive, tight contact structures on closed Seifert fibered 3–manifolds with base S^2, three singular fibers and $e_0 \geq 0$.

1 Introduction

The classification of positive, tight contact structures on lens spaces is due to K. Honda and E. Giroux [14, 11]. Let M be a closed, small Seifert fibered 3–manifold which is not a lens space. Then, M has base S^2 and exactly three singular fibers. Equivalently, M is orientation–preserving diffeomorphic to $M(r_1, r_2, r_3)$ for some $r_1, r_2, r_3 \in \mathbb{Q} \setminus \mathbb{Z}$, where $M(r_1, r_2, r_3)$ denotes the oriented 3–manifold given by the surgery diagram of Figure 1.
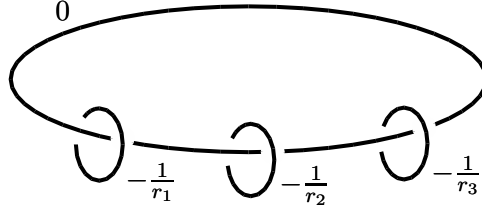

Figure 1: Surgery diagram for the Seifert fibered 3–manifold $M(r_1, r_2, r_3)$

Applying Rolfsen twists to the diagram of Figure 1 it is easy to show that

$$M(r_1, r_2, r_3) = M(r_1 + h, r_2 + k, r_3 - h - k), \quad h, k \in \mathbb{Z}. \qquad (1.1)$$

The integer

$$e_0(M(r_1, r_2, r_3)) := \sum_{i=1}^{3} [r_i]$$

is an invariant of the Seifert fibered 3–manifold $M(r_1, r_2, r_3)$.

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Recently H. Wu obtained the classification up to isotopy of positive tight contact structures on $M(r_1, r_2, r_3)$ (and therefore on small Seifert 3–manifolds) assuming $e_0 \neq -2, -1, 0$ [20]. He used convex surface theory to derive an upper bound for the number of isotopy classes of tight contact structures, and produced Legendrian surgery diagrams which show that the upper bound found in the first step is sharp.

In this note we extend Wu's results to the case $e_0 = 0$. More precisely, we classify positive tight contact stuctures on $M(r_1, r_2, r_3)$ assuming $e_0 \geq 0$, by using a set of Legendrian surgery diagrams which is slightly different from the one used in [20].

Observe that if $e_0(M(s_1, s_2, s_3)) \geq 0$, then by (1.1) we have

$$M(s_1, s_2, s_3) = M(r_1, r_2, r_3) \quad \text{for some} \quad r_1 > 0 \quad \text{and} \quad 1 > r_2, r_3 > 0.$$

For each of the three rational numbers r_1, r_2, r_3, we can write

$$-\frac{1}{r_i} = [a_0^1, a_0^2, \ldots, a_{n_i}^i] := a_0^i - \cfrac{1}{a_1^i - \cfrac{1}{\ddots - \cfrac{1}{a_{n_i}^i}}}, \quad i = 1, 2, 3,$$

for some uniquely determined integer coefficients

$$a_0^1 \leq -1 \quad \text{and} \quad a_0^2, a_0^3, a_1^i, \cdots, a_{n_i}^i \leq -2.$$

We define

$$T(r_1, r_2, r_3) := |(\prod_{i=1}^{3}(a_0^i + 1) - \prod_{i=1}^{3} a_0^i) \prod_{i=1}^{3} \prod_{k=1}^{n_i}(a_k^i + 1)|.$$

The following is our main result.

Theorem 1.1 *Suppose $r_1 > 0$ and $1 > r_2, r_3 > 0$. Then, $M = M(r_1, r_2, r_3)$ carries exactly $T(r_1, r_2, r_3)$ positive tight contact structures up to isotopy. Moreover, each tight contact structure on M has a Stein filling whose underlying 4–manifold has the handlebody decomposition given by Figure 2.*
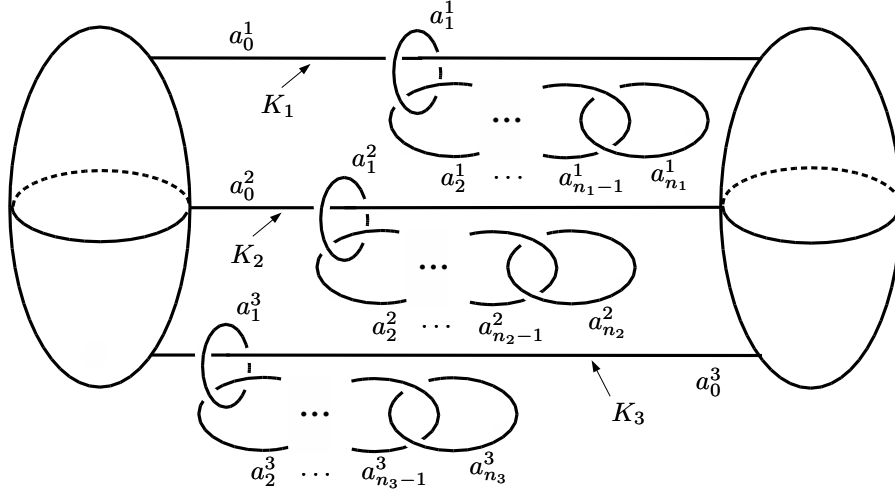
Figure 2: Handlebody decomposition of Stein fillings of $M(r_1, r_2, r_3)$

2   Upper bounds

In this section we establish an upper bound on the number of isotopy classes of
tight contact structures on the Seifert fibered 3–manifold $M = M(r_1, r_2, r_3)$.
Let ξ be a tight contact structure on M. Then, a Legendrian knot in M
smoothly isotopic to a regular fiber admits two framings: one coming from the
fibration and the other one coming from the contact structure ξ. The difference
between the contact framing and the fibration framing is the *twisting number*
of the Legendrian curve. We say that ξ has *maximal twisting equal to zero*
if there is a Legendrian knot isotopic to a regular fiber and having twisting
number zero.

Proposition 2.1 ([19], Theorem 1.3) *If $r_1, r_2, r_3 > 0$, then any tight contact
structure on $M(r_1, r_2, r_3)$ has maximal twisting equal to zero.* □

We can give an explicit construction of the Seifert manifold $M(r_1, r_2, r_3)$ as
follows. Let Σ be an oriented pair of pants, and identify each connected com-

ponent of
$$-\partial(\Sigma \times S^1) = T_1 \cup T_2 \cup T_3$$
with $\mathbb{R}^2/\mathbb{Z}^2$, so that $\binom{1}{0}$ gives the direction of $\partial(\Sigma \times \{1\})$ and $\binom{0}{1}$ gives the direction of the S^1 factor. Then glue a solid torus $D^2 \times S^1$ to each T_i using the map $\varphi_{A_i} : \partial(D^2 \times S^1) \to T_i$ defined by the matrix
$$A_i = \begin{pmatrix} \alpha_i & \alpha_i' \\ -\beta_i & -\beta_i' \end{pmatrix},$$
where
$$\frac{\beta_i}{\alpha_i} = r_i, \quad \alpha_i'\beta_i - \alpha_i\beta_i' = 1, \quad \text{and} \quad 0 < \alpha_i' < \alpha_i.$$
Since $r_i > 0$, it follows that $\beta_i > 0$. The singular fibers of the Seifert fibration will be denoted by F_i, $i = 1, 2, 3$.

Lemma 2.2 *Let $r_1, r_2, r_3 > 0$, and let ξ be a tight contact structure on $M = M(r_1, r_2, r_3)$. Then, the singular fibers F_i can be made Legendrian with twisting number -1. Moreover, there exist convex neighbourhoods U_i of F_i such that $-\partial(M \setminus U_i)$ has infinite slope.*

Proof Using Proposition 2.1, one can isotope ξ until there is a Legendrian regular fiber L with twisting number 0. Then, one can make the singular fibers F_i Legendrian with very low twisting numbers $n_i < 0$. Let V_i be a standard convex neighbourhood of F_i, and let A_i be a convex vertical annulus between L and a ruling of $\partial(M \setminus V_i)$. By the Imbalance Principle [14, Proposition 3.17], A_i produces a bypass attached to ∂V_i along a Legendrian curve with slope
$$-\frac{\alpha_i}{\alpha_i'} < -1.$$
Using the Twisting Number Lemma [14, Lemma 4.4] we can increase the twisting number of F_i up to -1. Then, we can thicken V_i further in order to obtain a convex solid torus U_i such that $-\partial(M \setminus U_i)$ has infinite slope. $\square$

Let Σ be a pair of pants. We say that a tight contact structure ξ on $\Sigma \times S^1$ is *admissible* if there is no contact embedding $(T^2 \times I, \xi_\pi) \hookrightarrow (\Sigma \times S^1, \xi)$, where ξ_π is a tight contact structure with convex boundary and twisting π (see [14, § 2.2.1] for the definition of twisting).

Lemma 2.3 *Let ξ be a tight contact structure on $M = M(r_1, r_2, r_3)$, and suppose that the singular fibers F_i are all Legendrian with twisting number -1. Let V_i be a standard neighbourhood of F_i, for $i = 1, 2, 3$. Then, the restriction of ξ to $M \setminus (V_1 \cup V_2 \cup V_3)$ is admissible.*

Proof Arguing by contradiction, suppose that

$$(T^2 \times I, \xi_\pi) \subset (M \setminus (V_1 \cup V_2 \cup V_3), \xi).$$

Any embedded torus in $M \setminus (V_1 \cup V_2 \cup V_3)$ contains a homotopically nontrivial embedded circle C which bounds a disc D in M. Since $(T^2 \times I, \xi_\pi)$ has twisting π, by [14, Proposition 4.16] there is a standard torus $T \subset T^2 \times [0, 1]$ with Legendrian divides isotopic to C. Then, D is isotopic to an overtwisted disc in (M, ξ). □

Lemma 2.4 *Let Σ be a pair of pants and ξ_+, ξ_- admissible tight contact structures on $\Sigma \times S^1$. Suppose that the boundary*

$$-\partial(\Sigma \times S^1) = T_1 \cup T_2 \cup T_3$$

consists of tori in standard form, with $\#\Gamma_{T_i} = 2$, $i = 1, 2, 3$, and slopes $s_1 = s_2 = s_3 = -1$. Let $\Sigma' \subset \Sigma$ be another pair of pants such that

$$\Sigma \times S^1 = \Sigma' \times S^1 \cup (T_1 \times I) \cup (T_2 \times I) \cup (T_3 \times I),$$

where $\xi_+|_{T_i \times I}$ and $\xi_-|_{T_i \times I}$ are, respectively, a positive and a negative basic slice with boundary slopes -1 and ∞. Then, $(\Sigma \times S^1, \xi_+)$ is isotopic to $(\Sigma \times S^1, \xi_-)$.

Proof By [9, Lemma 4.13] we can find vertical annuli $A_\pm \subset \Sigma \times S^1$ between T_1 and T_2 such that

(1) $A_\pm$ is convex and has Legendrian boundary with respect to $\xi_\pm$

(2) the dividing set of $A_\pm$ has no boundary parallel curves.

In fact, it is easy to check that A_- and A_+ are isotopic. Let ϕ_t be an isotopy of $\Sigma \times S^1$ which is the identity on the boundary and such that $\phi_1(A_-) = A_+$. To prove the lemma it suffices to show that ξ_+ is isotopic to $(\phi_1)_*(\xi_-)$. Therefore, without loss of generality we may assume $A_+ = A_- =: A$.

After rounding the edges,
$$(\Sigma \times S^1 \setminus A, \xi_\pm)$$

is isomorphic to a tight solid torus $(T^2 \times [0, 2], \eta_\pm)$, in such a way that $T^2 \times [0, 1]$ corresponds to

$$\Sigma \times S^1 \setminus (T_3 \times I \cup A)$$

and $T^2 \times [1, 2]$ corresponds to $T_3 \times I$. With this identification, the slopes of $T^2 \times \{0\}$ and $T^2 \times \{1\}$ are, respectively, -1 and $+1$. Also, notice that $(T^2 \times [0, 2], \eta_\pm)$ is minimally twisting because $(\Sigma \times S^1, \xi_\pm)$ is admissible.

A relative Euler class computation as in the proof of [9, Lemma 4.13] shows that

$$(T^2 \times [0,1], \eta_\pm|_{T^2 \times [0,1]}) \quad \text{and} \quad (T_1 \times [0,1], \xi_\pm|_{T_1 \times [0,1]})$$

are basic slices with the same sign. Similarly,

$$(T^2 \times [1,2], \eta_\pm|_{T^2 \times [1,2]}) \quad \text{and} \quad (T_3 \times [0,1], \xi_\pm|_{T_3 \times [0,1]})$$

have opposite signs, because the diffeomorphism between $T_3 \times \{0\}$ and $T^2 \times \{2\}$ reverses orientations. Thus, $(T^2 \times [0,2], \eta_+)$ and $(T^2 \times [0,2], \eta_-)$ both decompose into a positive basic slice and a negative basic slice belonging to the same continued fraction block, and therefore by [14, Lemma 4.14] they are isotopic. We conclude that $(\Sigma \times S^1, \xi_+)$ and $(\Sigma \times S^1, \xi_-)$ are isotopic because they decompose into pairwise isotopic pieces. $\qquad\square$

Lemma 2.5 *The following equalities hold for every $i = 1, 2, 3$:*

$$-\frac{\alpha_i}{\alpha_i'} = [a_{n_i}^i, \ldots, a_0^i] \tag{2.1}$$

$$-\frac{\alpha_i + (a_0^i + 1)\beta_i}{\alpha_i' + (a_0^i + 1)\beta_i'} = [a_{n_i}^i, \ldots, a_1^i + 1] \tag{2.2}$$

Proof Equality (2.1) follows from [17, Lemma A4]. A straightforward computation gives

$$-\frac{\alpha_i + (a_0^i + 1)\beta_i}{-\alpha_i - a_0^i \beta_i} = [a_1^i + 1, a_2^i, \ldots, a_{n_i}^i].$$

Thus, [17, Lemma A4] together with the fact that

$$(\alpha_i' + (a_0^i + 1)\beta_i')(-\alpha_i - a_0^i \beta_i) - (\alpha_i + (a_0^i + 1)\beta_i)(-\alpha_i' - a_0^i \beta_i') = 1$$

implies Equality (2.2). $\qquad\square$

Lemma 2.6 *Suppose $a_0^1 < -1$. Then, for every $i = 1, 2, 3$, the slope of the border between the first (i.e. the outermost) and the second continued fraction block of ∂U_i, computed in the basis of $-\partial(M \setminus U_i)$, is*

$$\frac{1}{a_0^i + 1}.$$

Proof According to [14, § 4.4.4] and in view of Equation (2.2), the slope of the border between the first and the second continued fraction block of ∂U_i is

$$[a_{n_i}^i, \ldots, a_1^i + 1] = -\frac{\alpha_i + (a_0^i + 1)\beta_i}{\alpha_i' + (a_0^i + 1)\beta_i}.$$

A direct computation via the matrix A_i gives the slope in the basis of $-\partial(M \setminus U_i)$. $\qquad\square$

Theorem 2.7 *Suppose $r_1 > 0$ and $1 > r_2, r_3 > 0$. Then, $M(r_1, r_2, r_3)$ carries at most*

$$T(r_1, r_2, r_3) := |(\prod_{i=1}^{3}(a_0^i + 1) - \prod_{i=1}^{3} a_0^i) \prod_{i=1}^{3} \prod_{k=1}^{n_i}(a_k^i + 1)|$$

distinct tight contact structures up to isotopy.

Proof Fix a decomposition of $M = M(r_1, r_2, r_3)$ as in Lemma 2.2:

$$M = M \setminus (U_1 \cup U_2 \cup U_3) \cup_{i=1}^{3} U_i.$$

Let $V_i \subset U_i$ be a standard neighborhood of the singular fiber F_i, for $i = 1, 2, 3$. Then, up to an isotopy which is fixed on the boundary there is at most one tight contact structure on $\cup_{i=1}^{3} V_i$. Moreover, by [7, Lemma 11], there is at most one admissible tight contact structure on $M \setminus (U_1 \cup U_2 \cup U_3)$ up to an isotopy not necessarily fixed on the boundary. Notice that, in general, one should allow only isotopies which are fixed on the boundary. But in our situation there is no loss in allowing more general isotopies on $M \setminus (U_1 \cup U_2 \cup U_3)$ because, as [8, Lemma 4.4.] shows, any isotopy on ∂U_i extends to U_i.

Let N_k^i be the $(k+1)$-th continued fraction block of $(U_i, \xi|_{U_i})$. The proof of Lemma 2.2 shows that the solid tori U_i have boundary slope

$$-\frac{\alpha_i}{\alpha_i'} < -1.$$

Therefore, by Equation (2.1) and [14, § 4.4.4] the slope of the border between N_{k-1}^i and N_k^i is $s_k^i = [a_{n_i}^i, \dots, a_k^i + 1]$. Thus, if we define

$$p_k^i := \#\{\text{positive basic slices in } N_k^i\},$$

we have

$$0 \le p_0^i \le |a_0^i + 1| \quad \text{and} \quad 0 \le p_k^i \le |a_k^i + 2|. \tag{2.3}$$

Let $V_i' := U_i \setminus N_0^i$. By Inequalities (2.3) and [14, Theorem 2.2], there are exactly

$$|(a_1^i + 1) \cdots (a_n^i + 1)|$$

distinct isotopy classes of tight contact structures on $V_i' \setminus V_i$, and $|a_0^1 a_0^2 a_0^3|$ possible configurations of signs (p_0^1, p_0^2, p_0^3) in $M \setminus (V_1' \cup V_2' \cup V_3')$. This immediately gives the number

$$|a_0^1 a_0^2 a_0^3 \prod_{i=1}^{3} \prod_{k=1}^{n_1}(a_k^i + 1)| \tag{2.4}$$

as an upper bound for the number of isotopy classes of tight contact structures on M. If $a_0^1 = -1$ (which is equivalent to $e_0(M) > 0$), clearly the quantity in (2.4) coincides with $T(r_1, r_2, r_3)$, and the statement follows.

Thus, we are left to prove the statement when $a_0^1 < -1$ (which is equivalent to $e_0(M) = 0$). In this case, the upper bound given by (2.4) is not optimal, because Lemma 2.4 shows that different sign configurations do not necessarily yield distinct contact structures on $M \setminus (V_1' \cup V_2' \cup V_3')$. In fact, if N_0^1, N_0^2 and N_0^3 contain basic slices B_1, B_2 and B_3 with the same sign, by [14, Lemma 4.14] we can arrange the basic slice decomposition of each N_0^i so that B_i is the first basic slice. Thus, it is easy to check using Lemma 2.6 that each B_i has boundary slopes -1 and ∞ when computed in the basis of $-\partial(M \setminus U_i)$. Applying Lemma 2.4, we are allowed to change the sign of all three basic slices simultaneously without changing the isotopy type of the contact structure. This shows that the configuration (p_0^1, p_0^2, p_0^3) is equivalent to $(p_0^1 \pm 1, p_0^2 \pm 1, p_0^3 \pm 1)$ (with the same signs chosen in each slot), whenever the sums are defined.

We can easily count the different possibilities for p_0^i: by the above argument we can always arrange that one of the p_0^i's is maximal, i.e. equal to $|a_0^i + 1|$. For the other two we have $|a_0^j| \cdot |a_0^k|$ many choices (where $\{i, j, k\} = \{1, 2, 3\}$). A simple computation shows that the total number of possibilities is equal to

$$|a_0^1| \cdot |a_0^2| + |a_0^2| \cdot |a_0^3| + |a_0^3| \cdot |a_0^1| - |a_0^1| - |a_0^2| - |a_0^3| + 1,$$

and this expression is equal to

$$|(\prod_{i=1}^3 (a_0^i + 1) - \prod_{i=1}^3 a_0^i)|.$$

This proves the statement when $a_0^1 < -1$, and concludes the proof. $\square$

3 Lower bounds

In this section we construct $T(r_1, r_2, r_3)$ distinct isotopy classes of Stein fillable, hence tight, contact structures on $M = M(r_1, r_2, r_3)$ assuming $r_1, r_2, r_3 > 0$.

Notice that the diagram of Figure 2 gives the handlebody decomposition of a 4-manifold X with boundary diffeomorphic to M. The decomposition involves a single 1–handle and some 2–handles.

Since $a_0^i \leq -1$ and $a_k^i \leq -2$ for $k > 0$, following [12] it is easy to describe a Stein structure on X by putting the knots into Legendrian position and stabilizing them until the prescribed framing coefficient becomes -1 with respect to their contact framing. This way we get

$$|\prod_{i=1}^3 \prod_{k=0}^{n_i} (a_k^i + 1)|$$

different Legendrian diagrams giving Stein structures on X and therefore tight contact structures on $\partial X = M$. Let us denote by ξ_J the contact structure corresponding to a Stein structure J on X. According to [16], if $c_1(J_1) \neq c_1(J_2)$ then the induced contact structures ξ_{J_1} and ξ_{J_2} are nonisotopic. Our aim is to count the number of distinct first Chern classes obtainable in this way.

In order to do this, we start by fixing a basis of $H_2(X;\mathbb{Z})$. We will present the second homology group using cellular homology. It is well–known [13] that the framed knots in the diagram correspond to 2–cells. Hence, a choice of orientation for the knots gives rise to a basis for the group $C_2(X)$ of 2–chains for X.

Let $C_1(X)$ denote the group generated by the 1–cells, i.e. by the 1–handles in the handle decomposition. Since there are no 3–handles present in the handle decomposition, $H_2(X;\mathbb{Z})$ can be computed as the kernel of the map $\varphi \colon C_2(X) \to C_1(X)$, given on a basis element $K \in C_2(X)$ corresponding to the knot K as

$$\varphi(K) = \sum a_i L_i,$$

where L_i runs through all 1–handles and $a_i \in \mathbb{Z}$ is the algebraic number of times K passes through the 1–handle L_i. In our case we have $C_1(X) \cong \mathbb{Z}$ and, for a suitable choice of orientations,

$$\varphi(K_i) = 1 \quad \text{for} \quad i = 1, 2, 3,$$

where the knots K_i are indicated in Figure 2, and φ is zero on each basis element $K \neq K_1, K_2, K_3$. Consequently, a basis of $H_2(X;\mathbb{Z})$ can be given by the homology classes corresponding to the unknots of Figure 2 together with the classes of

$$K_1 - K_2, K_1 - K_3 \in C_2(X).$$

It follows from the results of [12] that if $K \neq K_1, K_2, K_3$, then

$$\langle c_1(J), [K] \rangle = \mathrm{rot}(K),$$

while if $\{i, j\} = \{1, 2\}$ or $\{i, j\} = \{1, 3\}$,

$$\langle c_1(J), [K_i - K_j] \rangle = \mathrm{rot}(K_i) - \mathrm{rot}(K_j).$$

Theorem 1.1 follows immediately from Theorem 2.7 together with the following

Proposition 3.1 *Suppose $r_1, r_2, r_3 > 0$. Then, $M(r_1, r_2, r_3)$ carries at least*

$$T(r_1, r_2, r_3) := |(\prod_{i=1}^{3}(a_0^i + 1) - \prod_{i=1}^{3} a_0^i) \prod_{i=1}^{3} \prod_{k=1}^{n_i} (a_k^i + 1)|$$

distinct Stein fillable contact structures up to isotopy.

Proof Let J_1 and J_2 be Stein structures on X resulting from oriented Legendrian surgery diagrams as above. Denote by $r_i^k(1)$ and $r_i^k(2)$ ($i = 1, 2, 3$, $k = 0, \ldots, n_i$) the rotation numbers of the Legendrian knots appearing in the two diagrams. It follows from the above discussion that if either

$$r_1^0(1) - r_2^0(1) \neq r_1^0(2) - r_2^0(2), \quad r_1^0(1) - r_3^0(1) \neq r_1^0(2) - r_3^0(2),$$

or

$$r_i^k(1) \neq r_i^k(2) \quad \text{for some} \quad k > 0,$$

then $c_1(J_1) \neq c_1(J_2)$, and therefore by [16] the induced contact structures ξ_1 and ξ_2 on $\partial X = M$ are not isotopic. The conclusion follows from a computation similar to the one given in the proof of Theorem 2.7. □

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